

## CSci 1302 Assignment 6

Due Monday, Oct. 25th

**Problem 1 (5 points)** Of the following five formulas, which ones have the same meaning as the formula below?

$$\exists x.\forall y.p(x, y) \vee \exists z.p(z, y) \wedge p(w, z)$$

Please use notations of Example 7.2 on p. 99 to show which variable is bound to which quantifier. If a formula is not equivalent to the one above, please explain why.

- $\exists z.\forall x.p(z, x) \vee \exists x.p(x, x) \wedge p(w, x)$
- $\exists y.\forall z.p(y, z) \vee \exists x.p(x, z) \wedge p(w, x)$
- $\exists x.\forall y.p(x, y) \vee \exists x.p(x, y) \wedge p(w, x)$
- $\exists w.\forall x.p(w, x) \vee \exists y.p(y, x) \wedge p(w, y)$
- $\forall x.\exists y.p(y, x) \vee \exists z.p(z, x) \wedge p(w, z)$

**Problem 2 (24 points: 1,2,3 are 2 points each, the rest are 3 points each).** Assume the following:

1. A chess team A consists of Adam, Alice, and Ann.  $A(x)$  means that the person  $x$  is on the team A.
2. A chess team B consists of Bob and Beth.  $B(x)$  means that the person  $x$  is on the team B.
3. The relation  $wonAgainst(x,y)$  means that  $x$  has won against  $y$  at least once. Some people never played against each other, so no comparison is given for such pairs. The following are true statements:
  - (a) Adam has won against Ann and Alice.
  - (b) Alice has won against Ann.
  - (c) Beth has won against Adam and Bob.
  - (d) Bob has won against Alice.
  - (e) Ann has won against Alice.

Based on the information above, are the following true or false statements? Prove your answers by using tables. You may show only a part of the table if it is sufficient to prove or disprove the statement.

1.  $\exists x.A(x) \Rightarrow \text{wonAgainst}(x, \text{Beth})$
2.  $\forall x.\text{wonAgainst}(\text{Adam}, x) \vee \text{wonAgainst}(\text{Beth}, x)$
3.  $\forall x.B(x) \vee \neg \text{wonAgainst}(x, \text{Beth})$
4.  $\forall x.\exists y.A(x) \Rightarrow \text{wonAgainst}(y, x)$
5.  $\forall x.\exists y.A(x) \wedge \text{wonAgainst}(y, x)$
6.  $\exists x.\forall y.\text{wonAgainst}(x, y) \vee \text{wonAgainst}(y, x)$
7.  $\exists x.\exists y.\text{wonAgainst}(x, y) \wedge \text{wonAgainst}(\text{Adam}, x)$
8.  $\forall x.\forall y.A(x) \vee B(y)$
9.  $\forall x.\exists y.\neg \text{wonAgainst}(x, y)$

**Problem 3 (5 points)** Use the system in Problem 2 (without adding any new relations) and give an example of  $p(x, y)$  such that  $\forall x.\exists y.p(x, y)$  is true, but  $\exists y.\forall x.p(x, y)$  is not.