CSci 1302 Assignment 3
Due Wedn., September 29th, 2004

Note: the symbol $<\equiv>$ stands for logical equivalence.

Problem 1 (20 points). Prove the following:

1. $(p \Rightarrow q) \vee (p \Rightarrow r) \vee p$ is a tautology,

2. $(p \Rightarrow q) \wedge (p \Rightarrow r) \wedge p <\equiv> p \wedge q \wedge r$,

3. $(p \Rightarrow q) \wedge \neg q <\equiv> \neg(p \vee q)$,

4. $p \vee (q \wedge r \wedge s) <\equiv> (p \vee q) \wedge (p \vee r) \wedge (p \vee s)$,

5. $(p \wedge q \wedge r) \vee s <\equiv> (p \vee s) \wedge (q \vee s) \wedge (r \vee s)$.

Try to make your proof short. Proofs that are too long (i.e. have unnecessary steps) may get lower grades.

Problem 2 (15 points total, see points for individual questions below).
Consider a connective "exclusive OR", denoted by $\oplus$.

Question 1 (5 points). Using the symbol $\oplus$, write down the following properties:

1. idempotence of $\oplus$

2. commutativity of $\oplus$

3. associativity of $\oplus$

4. distributivity of $\oplus$ over $\wedge$

5. distributivity of $\Rightarrow$ over $\oplus$

You don't need to worry about whether these properties are true or false. You don't even need to know (yet) what $\oplus$ stands for.

Question 2 (5 points). $\oplus$ is defined as follows: $p \oplus q$ if and only if $(p \vee q) \wedge \neg(p \wedge q)$ (meaning, p or q, but not both). Rewrite the properties 1, 2, and 3 above without using the notation $\oplus$.

Question 3 (5 points). For the first two properties in Question 2 (idempotence and commutativity of $\oplus$) either give a transformational proof, or show that the property doesn't hold.

Extra credit, 4 points. Rewrite properties 4 and 5 without using the notation $\oplus$. You don't need to prove or disprove the properties.