

CSci 1302 Assignment 10

Due Fri., Dec. 3rd, 2004

Notations: \emptyset stands for the empty set, \mathbb{N} is the set of natural numbers $(1, 2, 3, \dots)$. $\mathbb{P}X$ stands for the power set of X .

Problem 1 (10 points). Let $X = \{x, y, z\}$ and $U = \{x, y, z, w, v\}$. For each of the statements below please indicate whether it is true, false, or does not make sense (for the “does not make sense” answer briefly explain why).

1. $x \in X$
2. $\{z, x\} \subseteq X$
3. $\{w\} \in X$
4. $\emptyset \in X$
5. $\emptyset \subset X$
6. $X \subset \emptyset$
7. $X \in \mathbb{P}X$
8. $\{y, w\} \in \mathbb{P}X$
9. $\emptyset \in \mathbb{P}X$
10. $\{\{x\}, \{x, y\}\} \subseteq \mathbb{P}X$
11. **Extra credit (1 point):** $\emptyset \subseteq \mathbb{P}X$.

Problem 2 (4 points). Given sets $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8, 10\}$, and the universal set $U = \{n \in \mathbb{N} \mid n \leq 10\}$, compute the following:

1. $A - B$
2. $A \cap B'$
3. $B' \cup A'$
4. $\mathbb{P}(A \cap B)$

Problem 3 (4 points). Consider the following sets (where $U = \mathbb{N}$):

- $A = \{n \in \mathbb{N} \mid \exists k.n = k^2\}$
- $B = \{n \in \mathbb{N} \mid \exists k.n = k^4\}$
- $C = \{n \in \mathbb{N} \mid \text{even}(n)\}$

Compute the following sets. **Important:** Justify your answers using propositional logic.

1. $A \cap B$

2. $A \cup B$
3. $C' \cap A$
4. $C' \cup (B - A)$

Problem 4 (10 points). Prove the following statements. A and B are arbitrary sets.

1. $A - U = \emptyset$
2. $(A')' = A$
3. $(A \cap B) \cup (A \cap B') = A$
4. $(A \cup B)' \cap A = \emptyset$
5. $(A \cup B)' \subseteq (A \cap B)'$