CSci 1302 Assignment 8

Due Mon., November 10, 2003

Problem 1 (15 points). Prove the following by induction. Make sure to read the claims carefully. It might be helpful to write out and check the first few cases before proving the general formula.

1. $\sum_{i=0}^{n-1}(2 \cdot i + 1) = n^2$ (note that the summation starts at $i = 0$, not $i = 1$).
2. $\sum_{i=1}^{n}(i \cdot 2^i) = (n-1) \cdot 2^{n+1} + 2$.
3. $4^n - 1$ is divisible by 3 for $n \geq 0$.

Problem 2 (5 points). Prove the following property of Fibonacci numbers by induction on n (m remains constant) $F_{n+m} = F_m \cdot F_{n+1} + F_{m-1} \cdot F_n$, assuming that $n \geq 0$ and $m \geq 1$.

Problem 3 (5 points). Consider a currency consisting of 2-cent and 5-cent coins only. Show by induction that any amount above 3 cents can be represented using these coins.

Hint: suppose you are given a bag of coins whose sum is n ($n > 3$). How can you replace some coins in the bag to increment the total amount in the bag by 1?