CSci 1302 Assignment 6
Due Wedn., October 22, 2003

This problem set uses some of the following predicates on integer numbers:

- Unary: \(\text{prime}(x), \text{even}(x), \text{odd}(x)\) mean “\(x\) is prime”, “\(x\) is even”, “\(x\) is odd”, respectively. \(\text{thelargest}(x)\) means that \(x\) is the largest of all numbers.

- Binary: \(\text{equal}(x, y)\) means “\(x\) is equal to \(y\)”, \(\text{greater}(x, y)\) means “\(x > y\)”, \(\text{divisible}(x, y)\) means “\(x\) is divisible by \(y\)”.

- Ternary: \(\text{sum}(x, y, z)\) means that \(x = y + z\).

While the meaning of the predicates is not important for the proofs, it might be helpful in providing intuition about the statements.

**Problem 1 (16 points).** Prove the following arguments in the predicate logic. Use subscripts (such as \(x_1\) and \(x_2\)) to distinguish between unknown and genuine variables after instantiation.

1. \(\forall x.\text{even}(x) \iff \text{divisible}(x, 2)\)
   \[\exists x.\text{even}(x)\]
   \[\exists x, \exists y.\text{divisible}(x, y)\]

2. \(\forall x, \forall y, \exists z.\text{sum}(x, y, z)\)
   \[\exists x, \exists y.\text{sum}(x, y)\]

3. \(\forall x, \forall y, \text{greater}(x, y) \lor \text{greater}(y, x)\)
   \[\forall x, \neg \text{greater}(x, x)\]
   \[\text{false}\]

4. \(\forall x, \text{thelargest}(x) \Rightarrow \forall y, \text{greater}(x, y)\)
   \[\exists w, \forall z, \text{greater}(w, z)\]
   \[\forall x, \neg \text{thelargest}(x)\]

**Problem 2 (8 points).** Exercise 9.4 p. 131, parts 2,3 only. Use the same notations as in problem 1.

**Problem 3 (8 points).** Exercise 9.5 p. 132, parts 1 and 2 only.