

## CSci 1302 Assignment 6

Due Wedn., October 22, 2003

This problem set uses some of the following predicates on integer numbers:

- Unary:  $prime(x)$ ,  $even(x)$ ,  $odd(x)$  mean “ $x$  is prime”, “ $x$  is even”, “ $x$  is odd”, respectively.  $thelargest(x)$  means that  $x$  is the largest of all numbers.
- Binary:  $equal(x, y)$  means “ $x$  is equal to  $y$ ”,  $greater(x, y)$  means “ $x > y$ ”,  $divisible(x, y)$  means “ $x$  is divisible by  $y$ ”.
- Ternary:  $sum(x, y, z)$  means that  $x = y + z$ .

While the meaning of the predicates is not important for the proofs, it might be helpful in providing intuition about the statements.

**Problem 1 (16 points).** Prove the following arguments in the predicate logic. Use subscripts (such as  $x_{\exists}$  and  $x_{\forall}$ ) to distinguish between unknown and genuine variables after instantiation.

1.  $\forall x.even(x) \Leftrightarrow divisible(x, 2)$   
 $\exists x.even(x)$   
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 $\exists x.\exists y.divisible(x, y)$
2.  $\forall x.\forall y.\exists z.sum(x, y, z)$   
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 $\forall x.\exists y.sum(x, x, y)$
3.  $\forall x.\forall y.greater(x, y) \vee greater(y, x)$   
 $\forall x.\neg greater(x, x)$   
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**false**
4.  $\forall x.thelargest(x) \Rightarrow \forall y.greater(x, y)$   
 $\neg \exists w.\forall z.greater(w, z)$   
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 $\forall x.\neg thelargest(x)$

**Problem 2 (8 points).** Exercise 9.4 p. 131, parts 2,3 only. Use the same notations as in problem 1.

**Problem 3 (8 points).** Exercise 9.5 p. 132, parts 1 and 2 only.