CSci 1302 Assignment 5

Due Fri., October 10, 2003

This problem set uses the following predicates on integer numbers:

- Unary: $prime(x), even(x), odd(x)$ mean "x is prime", "x is even", "x is odd", respectively.
- Binary: $equal(x, y)$ means "x is equal to y", $greater(x, y)$ means "$x > y$", $divisible(x, y)$ means "x is divisible by y".
- Ternary: $sum(x, y, z)$ means that $x = y + z$.

Problem 1 (24 points). Write the following formulas in English. For each claim say whether it is true or false and briefly explain your answer.

1. $\forall x.\exists y.sum(x, y, y)$
2. $\forall x.\exists y.sum(y, x, x)$
3. $\exists x.\forall y.sum(x, x, y)$
4. $\forall x.even(x) \wedge \exists y.sum(x, y, y)$
5. $\forall x.odd(x) \Rightarrow \forall y.\neg sum(x, y, y)$
6. $\forall x.\forall y.\neg equal(x, y) \Rightarrow \forall z.divisible(x, z) \Rightarrow \neg divisible(y, z)$
7. $\forall x.\forall y.divisible(x, y) \Rightarrow \exists z.greater(z, x) \wedge divisible(z, y)$
8. $\forall x.\exists y.\forall z.greater(z, x) \Rightarrow divisible(z, y)$

Problem 2 (16 points). Write the following sentences as formulas in predicate logic.

1. Not all odd numbers are prime.
2. Not all prime numbers are odd.
3. All even numbers greater than 2 are divisible by 4.
4. No even number is greater than itself.
5. There is a number which is smaller than all even numbers.
6. There is the smallest even number.
7. Every even number can be represented as the sum of two odd numbers.
8. If a number is even then it is divisible by some number other than itself.

Problem 3 (10 points total).

Question 1 (8 points) For each of the following formulas please show which variables are bound and which ones are free. For all bound variables show which quantifier they are bound to. Use notations of Example 7.2 on p. 99.

1. $\exists x.p(x, y) \land (\forall y.p(x, y)) \land \forall z.q(z, y) \Rightarrow r(z)$
2. $\exists z.p(z, y) \land (\forall z.p(z, z)) \land \forall x.q(x, y) \Rightarrow r(x)$
3. $\exists z.p(z, y) \land (\forall x.p(z, x)) \land \forall s.q(s, y) \Rightarrow r(s)$
4. $\exists y.p(y, x) \land (\forall z.p(y, z)) \land \forall y.q(y, x) \Rightarrow r(y)$

Question 2 (2 points) Which of the formulas 2-4 in Question 1 are equivalent to the first one?