

CSci 1302 Assignment 3
Due Wedn., September 24, 2003

Note: the symbol $\langle \equiv \rangle$ stands for logical equivalence.

Problem 1 (20 points). Prove the following:

1. $(p \Rightarrow q) \vee (p \Rightarrow r) \vee p$ is a tautology,
2. $(p \Rightarrow q) \wedge (p \Rightarrow r) \wedge p \langle \equiv \rangle p \wedge q \wedge r$,
3. $(p \Rightarrow q) \wedge \neg q \langle \equiv \rangle \neg(p \vee q)$,
4. $p \vee (q \wedge r \wedge s) \langle \equiv \rangle (p \vee q) \wedge (p \vee r) \wedge (p \vee s)$,
5. $(p \wedge q \wedge r) \vee s \langle \equiv \rangle (p \vee s) \wedge (q \vee s) \wedge (r \vee s)$.

Try to make your proofs as short as possible. Proofs that are too long may get lower grades.

Problem 2 (20 points). Consider a connective exclusive OR, denoted by \oplus .

Question 1 (5 points). Using the symbol \oplus , write down the following properties:

1. idempotence of \oplus
2. commutativity of \oplus
3. associativity of \oplus
4. distributivity of \oplus over \wedge
5. distributivity of \Rightarrow over \oplus

You don't need to worry about whether these properties are true or false. You don't even need to know (yet) what \oplus stands for.

Question 2 (10 points). Recall that \oplus is defined as follows: $p \oplus q$ if and only if $(p \vee q) \wedge \neg(p \wedge q)$.

Rewrite the properties above without using the notation for \oplus .

Question 3 (5 points + possible extra credit). For the first two properties in Question 2 (idempotence and commutativity of \oplus) either give a transformational proof, or show that the property doesn't hold. To show that a property **doesn't hold**, you may use truth tables. However, you will earn extra credit if you use a transformational proof to disprove the property (for instance, you may show that the right-hand side and the left-hand side reduce to simple formulas which are clearly not equivalent).