CSci 1302 Assignment 12

Due Wedn., December 10th, 2003

Notations: $\emptyset$ stands for the empty set, $\mathbb{N}$ is the set of natural numbers $(1, 2, 3, ...)$. $\mathbb{P}X$ stands for the power set of X.

Problem 1 (6 points). Please classify the following relations on natural numbers $\mathbb{N}$ as:
reflexive/irreflexive/non-reflexive,
symmetric/anti-symmetric/non-symmetric, and
transitive/non-transitive.

1. $R = \{(n, m) \mid n + m \text{ is even}\}$
2. $R = \{(n, m) \mid n + m \text{ is odd}\}$
3. $R = \{(n, m) \mid n \text{ is even}, m \text{ is even}\}$
4. $R = \{(n, m) \mid n \text{ is even}, m \text{ is odd}\}$ (think carefully about transitivity of this relation).

Problem 2 (6 points). Construct a reflexive, a symmetric, a transitive (R^+), and a reflexive transitive (R^*) closures of each of the following relations on the set $A = \{a, b, c, d\}$. You may represent the resulting relation as a picture instead of writing out the list of its elements.

1. $R = \{(a, b), (b, c), (c, d), (d, a)\}$,
2. $R = \{(a, a), (a, b), (a, d), (c, c), (c, b), (c, d)\}$.

Problem 3 (Extra Credit, 5 points) Let R be a relation. Consider the following two relations:

1. Suppose that R_r is the reflexive closure of R, R_s is the symmetric closure of R, and R^+ is the (non-reflexive) transitive closure of R. Let $\widehat{R} = R_r \cup R_s \cup R^+$.
2. Suppose that R_r is the reflexive closure of R, R'_s is the symmetric closure of R_r (i.e. of the reflexive closure of R, not of R itself), and $\overline{R}$ is the (non-reflexive) transitive closure of R'_s.

Are the two relations ($\widehat{R}$ and $\overline{R}$) equal for all possible relations R? Is any one of them guaranteed to be an equivalence relation? In case of a positive answer please justify, in case of a negative answer please give a counterexample.

You may earn a partial credit on this problem if you answer some, but not all, of the questions.

Problem 4 (4 points). Consider the following directed graph: $V = \{a, b, c, d\}$, $E = \{(a, b), (b, a), (b, b), (b, c), (b, d), (c, d)\}$.

1. draw the picture of the graph
2. write down the adjacency matrix of the graph

3. find all simple paths from a to c

4. find all cycles in the graph

Problem 5 (6 points). Consider the following undirected graph: $V = \{a, b, c, d\}$, $E = \{(a, b), (b, b), (b, c), (b, d), (c, d)\}$.

1. draw the picture of the graph

2. write down the adjacency matrix of the graph

3. find all simple paths from a to c

4. find all cycles in the graph

5. show the work of breadth-first search algorithm on this tree, starting from the vertex a. More specifically, show the sequence of states of the queue (the state changes when a vertex is put on the queue or removed from it) and the resulting marking of all vertices.

Problem 6, the very last one! (5 points). Consider the following relations from the set $A = \{a, b, c, d\}$ to the set $B = \{1, 2, 3, 4\}$:

1. $\{(a, 1), (b, 1), (c, 1)\}$

2. $\{(a, 1), (a, 2), (a, 3)\}$

3. $\{(a, 1), (b, 2), (c, 3), (d, 4), (a, 2)\}$

4. $\{(a, 1), (b, 2), (c, 3), (d, 1)\}$

5. $\{(a, 1), (b, 2), (c, 1), (d, 2)\}$

For each relation please say whether the relation is a function, and if it is, whether it is partial or total. Please explain your answer briefly.